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## Black Hole Information vs. Locality

N.Itzhaki <sup>1</sup>

Raymond and Beverly Sackler Faculty of Exact Sciences

School of Physics and Astronomy

Tel Aviv University, Ramat Aviv, 69978, Israel

### Abstract

We discuss the limitations on space time measurement in the Schwarzschild metric. We find that near the horizon the limitations on space time measurement are of the order of the black hole radius. We suggest that it indicates that a large mass black hole cannot be described by means of local field theory even at macroscopic distances and that any attempt to describe black hole formation and evaporation by means of an effective local field theory will necessarily lead to information loss. We also present a new interpretation of the black hole entropy which leads to  $S = cA$ , where  $c$  is a constant of order 1 which does not depend on the number of fields.

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<sup>1</sup>Email Address:sanny@post.tau.ac.il

## 1.Introduction

The connection between black hole area and statistical mechanics entropy is one of the most interesting question in physics. Black hole analogs of the laws of thermodynamics exist [1, 2] , with the area of a classical black hole playing the role of entropy. Furthermore , the generalized second law of thermodynamics [4, 5, 3] implies that the sum of the ordinary entropy and the black hole area (divided by 4 in units where  $G = \hbar = c = 1$ ) never decreases. Finally, black holes radiate at the temperature ,  $T = \frac{dM}{dS}$  where  $S = \frac{A}{4}$  [6, 7]. However, the statistical nature of the black hole entropy is still unclear. Another intricate related issue is the information loss puzzle [6]. There seem to be three principal alternatives:

1. The information is lost [6].
2. The information is stored in the correlation between Hawking radiation and Planck-mass remnants [8].
- 3.All the information is emitted with the Hawking radiation [9, 10, 11].

In this paper we study new aspects of the black hole puzzle. We consider a Gedanken experiment in which a macroscopic system falls into a black hole. We are interested in the *absolute* limitations on the information that an external observer can obtain on the system *before* it reaches the horizon . It has been argued by many authors [12, 13, 14, 15, 16, 17, 18, 19, 20] that Planck scale is the minimal observable scale. In that case one should expect to find absolute limitations only after a time larger then  $M \log(MR)$  (where  $R$  is

the initial location of the system) when the matter settles into a layer whose invariant distance from the horizon is of the order of 1. So the radial information is lost. Note that the angular information is not lost. This problem is closely related to the trans-Planckian frequencies problem and to 't Hooft diagnosis that conventional quantum fields contribute ultraviolet divergences to the entropy near the horizon; as such it cannot be investigated without further knowledge on quantum gravity <sup>2</sup>. Recently, we have shown [23] that there are states in which the uncertainties in space time measurements are much larger than Planck scale. This leads to problems with the conventional definition of statistical mechanics in quantum gravity even in the absence of black holes. In the presence of black holes this raises a new possibilities of information loss in Gedanken experiments which can be studied without further knowledge of quantum gravity.

## **2.Space-time measurements in Schwarzschild metric**

In this section we study the limitations on space-time measurements in a Schwarzschild background metric , especially near the horizon. First, we

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<sup>2</sup>Still one can learn on black holes from the fact that Planck scale is the minimal observable scale since then alternatives 1 and 2 are essentially the same concerning the available information. According to the remnants approach the information on the collapsing star which form the black hole is in the correlation between the structure of the remnant and Hawking radiation. However, since the size of the remnants is of the order of Planck scale one cannot measure the structure of the remnants so the information is lost in principle. In other words, although the state of the remnant and Hawking radiation is suppose to be a pure state. One cannot distinguish between different states of the remnants. One must, therefore, sum over all the possible states of the remnants so effectively one describes the state of Hawking radiation as a mixed state. We should note that this argument certainly does not support previous arguments [21, 22] that remnants lead to mathematical inconsistency .

would like to consider a distant external observer who measures the coordinate  $(t, R, \theta, \varphi)$  of some space-time event. In classical gravity the energy momentum tensor of the detector which is supposed to measure the location of the event can be as small as one wishes and therefore it does not affect the metric. So, as long as  $R > 2M$  there are no limitation in principle on space time measurements. For an external observer, the time it takes for a particle to reach the horizon is infinite, thus at any finite time one can measure the exact location and momenta of the particles which fall into the black hole. Thus in classical general relativity one can construct (at least in principle) a measuring device which follows the trajectory of the particles in phase space even when a black hole is present, so information is not lost in principle. The meaning of the no hair theorem in that context is that for practical purposes the in falling particles are suddenly cut off from communication with the external observer, since the red-shift grows exponentially in time near the horizon. The only information which is available in practice is the total mass, angular momentum and charge while in principle all the information is available.

In quantum gravity the problem is much more interesting. In the absence of a quantum theory of gravity, we need to introduce two postulates on quantum gravity in order to make the discussion possible.

The postulates are the following:

*Postulate 1.* At large distances the first order of the gravitational effect

in quantum gravity can be described to a good approximation by general relativity.

*Postulate 2.* At large distances quantum gravity is a local theory, meaning there are no non-local effects at large distances. We denote the minimal scale for which the postulates are correct as  $x_c$ . A few remarks are in order now. First, since the only fundamental scale in quantum gravity is  $l_p$  ( $l_p = \sqrt{\frac{G\hbar}{c^3}}$ ) it is natural to assume that  $x_c \approx l_p = 1$ . Second, these postulates are the easiest way to describe the correspondence principle, between quantum gravity and general relativity and between quantum gravity and local quantum field theory.<sup>3</sup> However, the postulates are not a general properties of quantum gravity since there are more complicated way to describe the correspondence principle. In section 4 we suggest that postulate 2 is incorrect and that  $x_c$  depends on the state of the system. Third, in almost any discussion on quantum gravity, one uses those postulates. In particular, the conventional argument for information loss rests on those postulates.

Let us focus first on the time measurement. Basically we follow [13] but in Schwarzschild metric instead of flat space-time. In order to measure  $t$  there must be a clock located at  $(R, \theta, \varphi)$  which emits at least one photon towards the external observer. There are two causes of error in this process of time measurement:

1- The uncertainty of the clock  $-\Delta t$ .

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<sup>3</sup>Moreover, this correspondence principle is the only experimental data that we have on quantum gravity.

2- The uncertainty in the time it takes for the photon to reach the external observer . This uncertainty is due to the uncertainty in the metric caused by the uncertainty of the energy of the clock  $\Delta E$ .

The time it takes for the photon to reach the external observer is

$$T = \int_R^X \frac{dr}{v} = (X - R) + 2M_{tot} \log\left(\frac{X - 2M_{tot}}{R - 2M_{tot}}\right), \quad (1)$$

where  $X$  is the location of the external observer and  $M_{tot}$  is the total mass of the black hole and the clock. So we get

$$\Delta T > 2\Delta E \frac{M_{b.h}}{\delta} \quad (2)$$

where  $\delta = R - 2M_{b.h}$ . Adding the quantum uncertainty, the uncertainty for the whole process of time measuring is therefore

$$\Delta T_{tot} > \frac{1}{\Delta E} + 2\Delta E \frac{M_{b.h}}{\delta} \geq \sqrt{\frac{8M_{b.h}}{\delta}}. \quad (3)$$

Note that the minimum is obtained for  $\Delta E \approx \sqrt{\frac{\delta}{M_{b.h}}}$ , thus near the horizon the clock is rather regular (no planckian uncertainty). This result is not surprising since the uncertainty in the invariant distance is of order one:  $\Delta T_{tot} \sqrt{g_{0,0}} \approx 1$ . The limitation on  $R$  measurement is

$$\Delta R_{tot} = \Delta T_{tot} \frac{\partial R}{\partial T}. \quad (4)$$

From Eq.(1) we get  $\frac{\partial R}{\partial T} = \frac{2M}{\delta}$  thus

$$\Delta R \geq \sqrt{\frac{\delta}{M}}. \quad (5)$$

Again this is not surprising since the uncertainty in the invariant distance is of order one:  $\Delta R \sqrt{g_{r,r}} \approx 1$ .

There are at least two different ways to measure  $\theta$  or  $\varphi$  which leads to the same surprising result. The first is based on the logic of Eq.(4) but with  $\varphi$  or  $\theta$  instead of  $R$ . The other way is more straight forward and is the following: Suppose that there is an apparatus which send a light signal from the event in the radial direction. Classically  $\frac{d\theta}{dt} = \frac{d\varphi}{dt} = 0$  so  $\theta_f = \theta$  and  $\varphi_f = \varphi$ . The external observer need therefore to measure only  $\theta_f$  and  $\varphi_f$  in order to know  $\theta$  and  $\varphi$ . Let us treat this process quantum mechanically. Consider the Schwarzschild metric

$$d\tau^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (6)$$

Where  $B(r) = 1 - \frac{2M}{r} = A(r)^{-1}$ . According to postulate 1 we can use this metric at scale larger than 1. The geodesic equations are [24]

$$r^2 \frac{d\varphi}{dp} = J \quad (\text{constant}) \quad (7)$$

$$A(r) \left( \frac{dr}{dp} \right)^2 + \frac{J^2}{r^2} - \frac{1}{B(r)} = -E \quad (\text{constant}), \quad (8)$$

where  $p$  is a parameter describing the trajectory and  $J$  is the angular momentum per unit mass. The connection between the proper time  $\tau$  and  $p$  is

$$d\tau^2 = E dp^2 \quad (9)$$

so we find that

$$E > 0 \quad \text{for material particles} \quad (10)$$

$$E = 0 \quad \text{for photons} \quad (11)$$

Since  $A(r)$  is positive the particle can reach a radius  $r$  only if

$$\frac{2J^2}{r^2} + E \leq \frac{1}{B(r)} \quad (12)$$

which leads to

$$J^2 \leq \frac{r^3}{r - 2M} \equiv F(r) \quad (13)$$

for photons. Let us find the upper limit on  $J$  such that the light signal will reach an external observer. The minimum of  $F(r)$  is  $27M^2$  at  $\hat{r} = 3M$ . Thus, in order that the photon will cross  $\hat{r}$  we must impose

$$J^2 \leq 27M^2. \quad (14)$$

Now, suppose that the photon has uncertainty  $\Delta\varphi$  at the emission point. From postulate 2 and the uncertainty principle we get

$$\Delta P_\varphi \geq \frac{1}{\Delta\varphi R}, \quad (15)$$

where  $P_\varphi$  is the momentum of the photon in the  $\varphi$  direction. In Minkowski space

$$V_\varphi = R \frac{d\varphi}{dt} = \frac{P_\varphi}{P}. \quad (16)$$

where  $P_\varphi$  is the momentum at the  $\varphi$  direction and  $P$  is the total momentum, so in Schwarzschild metric we get

$$\frac{d\varphi}{dt} = \frac{P_\varphi}{RP} \sqrt{B(R)}. \quad (17)$$



Since

$$J = \frac{R^2}{B(R)} \frac{d\varphi}{dt}, \quad (18)$$

we obtain

$$\Delta J = \frac{R\Delta P_\varphi}{P} \sqrt{\frac{1}{B(R)}}. \quad (19)$$

From Eqs.(7,8) one obtains [24]

$$\varphi_f = \varphi \pm \int \frac{A(r)^{\frac{1}{2}} dr}{r^2 \left( \frac{1}{J^2 B(r)} - \frac{1}{r^2} \right)^{\frac{1}{2}}} \quad (20)$$

The connection between the momentum of the photon and its total energy (including the gravitational energy of the interaction between the photon and the black hole) differs from the connection in Minkowski space by the red-shift factor.

$$E_\gamma = P \sqrt{B(r)}, \quad (21)$$

thus we get

$$\Delta\varphi \geq \frac{1}{\Delta P_\varphi R} + \frac{\Delta P_\varphi}{E_\gamma} \quad (22)$$

so,

$$\Delta\varphi_{min} \geq \frac{1}{\sqrt{M E_\gamma}} \quad (23)$$

Finally, Eq.(1) implies that the maximal energy of the photon is

$$E_\gamma \leq \delta \quad (24)$$

other wise the Schwarzschild radius of the black hole and the clock before the emission of the photon is larger then  $R$  . so we to get

$$\Delta\varphi_{min} \geq \frac{1}{\rho} \quad (25)$$

where  $\rho$  is the invariant distance from the horizon:

$$\rho = \int_{2M}^R ds = \int_{2M}^R \frac{dr}{\sqrt{1 - \frac{2M}{r}}} = \sqrt{8M(R - 2M)} \quad (26)$$

Notice that the minimum is obtained at  $\Delta J = \frac{M}{\rho}$ , so Eq.(14) does not play an important role in our discussion (according to the postulates the discussion is meaningful only for  $\rho > 1$ ) and the small perturbation approximation of Eq.(20) is valid.

The minimal uncertainty in the area of a sphere with invariant distance from the horizon  $\rho$  is ,

$$\Delta A \geq \frac{1}{\rho} A. \quad (27)$$

Notice that for  $\rho \approx 1$  we get  $\Delta A = A$ . This suggests that for external observer, all the angular information of the particles involved in the formation of the black hole is lost<sup>4</sup>. We should remark that for  $\rho > M$  one should consider the regular limitations on space time measurement [13] which are then non negligible and lead to  $\Delta A \approx 1$ .

Next, we turn to the measurement of the distance between two events which occur near the horizon of an infinitely massive black hole. In that limit the resulting geometry outside the event horizon is described by the Rindler metric.

$$d\tau^2 = dT^2 - dZ^2 - dX^i dX^i. \quad (28)$$

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<sup>4</sup>Except, of course, from the total angular momentum which can be measured by measuring the Kerr metric at large distance. Here we assume that the total angular is zero.

In terms of Schwarzschild coordinates it is given by

$$d\tau^2 = \left(\frac{dt}{4M}\right)^2 \rho^2 - \rho^2 - dX^i dX^i. \quad (29)$$

The Minkowski and Schwarzschild coordinates are related by

$$Z = \rho \cosh\left(\frac{t}{4M}\right)$$

$$T = \rho \sinh\left(\frac{t}{4M}\right). \quad (30)$$

We did not use Rindler metric in the distant observer discussion since Rindler metric is a good approximation to Schwarzschild metric only for  $\rho \ll M$ , while the invariant distance between the external observer and the horizon is at least of the order of  $M$ . In Minkowski space the minimal uncertainty in  $T$  is of order 1 [13], therefore

$$\Delta t \approx \frac{M}{\rho}, \quad (31)$$

which is in agreement with Eq.(3). Suppose that the two events ( $a$  and  $b$ ) occur at the same  $X^i$  and that one wishes to measure  $\rho = \rho_a - \rho_b$ . The measurement can be carried out in the following way: A clock at  $\rho_a$  measures the time  $t_i$  when a photon is sent towards  $\rho_b$ . At the other object there is a mirror which reflects the photon back to the first object, where the clock measures the time  $t_f$  when a photon arrives. It is easy to find that

$$t_f - t_i = 4M \ln\left(\frac{\rho_a}{\rho_b}\right) \quad (32)$$

Since there is a minimal uncertainty in  $t$  there is also a minimal uncertainty in  $\rho$ ,

$$\Delta\rho = \Delta t \frac{d\rho}{dt} \approx 1 \quad (33)$$

Suppose that the two events occur at the same  $\rho$  and that one wishes to measure the transverse distance  $X_t$ , where  $X_t^2 = (X_a^i - X_b^i)(X_a^i - X_b^i)$ . For  $X_t \gg \rho$  the time it takes for a light signal to travel from  $a$  to  $b$  is [25]

$$t = 8M \ln\left(\frac{X_t}{\rho}\right), \quad (34)$$

so

$$\Delta X_t \approx \frac{1}{\rho} X_t. \quad (35)$$

### 3. Information loss and black hole entropy

In field theories which do not involve gravitation there is information loss in practice -i.e. the number of orthogonal states with the same macroscopic properties can never decrease. In principle, however information is conserved, since in principle one can always construct a measuring device which measures the exact state of the system. Therefore, in field theories which do not involve gravitation

$$S_{mat} = S_{phy}. \quad (36)$$

Where  $S_{mat}$  is the log of the number of orthogonal states with the same macroscopic properties and  $S_{phy}$  is the log of the maximal number of orthogonal states with the same macroscopic properties which can be distinguished

by means of local measurements. Note that if one permits a non-local interaction between the measuring device and the system (such as a Von-Neumann interaction) then one can distinguish between all states and there is no information loss even in the presence of gravitation. However the interaction between the measuring device and the system as any other interaction must be local.

If postulates 1 and 2 are correct then in quantum gravity there are cases for which

$$S_{mat} > S_{phy} \quad (37)$$

In [23] we present such a case using the weak field approximation. We argued that the basic reason for information loss in quantum gravity is the locality postulate (postulate 2) and not the horizon ( in the weak field case there is obviously no horizon). The horizon , however , causes a strong red-shift which makes the information loss more obvious then in the weak field case. Let us demonstrate the information loss by considering a simple example: two orthogonal states of a particle which falls into a black hole .

$$\langle r, \theta, \varphi | \phi_1 \rangle = R(r) \Theta_1(\theta) \Psi(\varphi) \quad (38)$$

$$\langle r, \theta, \varphi | \phi_2 \rangle = R(r) \Theta_2(\theta) \Psi(\varphi) \quad (39)$$

where  $\Theta_1(\theta)$  yields  $\theta = \pm \Delta\theta$  and  $\Theta_2(\theta)$  yields  $\theta = \pi \pm \Delta\theta$  and  $\Delta\theta \ll \pi$ . Suppose that at  $t = 0$   $r \approx 3M$ , so that  $|\phi_1\rangle$  and  $|\phi_2\rangle$  are distinguishable by means of local interactions. After a finite time of the order of  $2M \log M$ , the

invariant distance between the particles and the horizon is of the order of one, then from Eq.(25) we learn that  $\Delta\theta = \pi$ , thus the external observer cannot distinguish between the two orthogonal states by means of local interactions.

Consider now a macroscopic system which at  $t = 0$  is located at a distance  $R$  ( $R > 3M$ ), so that  $S_{mat} = S_{phy}$ . After a finite time of the order of  $M \log RM$  the invariant distance between the object and the black hole is of order 1 and the object is in the volume  $0 \leq \rho \leq 1$ ,  $0 \leq \theta \leq \pi$ ,  $0 \leq \varphi \leq 2\pi$ . But the minimal uncertainties are  $\Delta\rho \approx 1$ ,  $\Delta\theta \approx \pi$  and  $\Delta\varphi \approx 2\pi$ . So one cannot distinguish between any orthogonal states with the same macroscopic properties. This agree with the classical no hair theorem. The difference is that in quantum theory unlike classical theory the absolute information loss is a smooth function of time and the total information loss takes a finite time. Since the black hole is made out of collapsing objects it implies that for black holes

$$S_{phy} = 0. \tag{40}$$

It is not surprising, therefore, that postulates 1 and 2 lead to a non-unitarity description of the formation and evaporation of black holes since as we found out information is lost in principle in the formation of a black hole within a finite time. There have been many attempts to shown that  $S_{mat} = S_{b.h}$  so Hawking radiation is just a statistical mechanics radiation. We find this idea disturbing since, if our arguments are correct, then one cannot distinguish in principle among the states which are counted in  $S_{mat}$ , so black hole entropy

cannot be defined as the log of the number of orthogonal states which can be distinguished in principle but cannot be distinguished in practice (the same macroscopic properties).

What is, then, the Bekenstein-Hawking entropy? Recall that the special property of gravitation which causes information loss is the fact that in general relativity, unlike in any other theory, the fields define distances. It is only natural, therefore, to define the locality loss entropy ( $S_{l,l}$ ) as the number of orthogonal states of the metric that cannot be distinguished in principle by means of local measurements. If there had not been limitations on space time measurement then  $S_{l,l}$  would have been zero, since any orthogonal states of the metric define different distances so in principle they are distinguishable in that case. However, there do exist limitations on space time measurements. Therefore, there are orthogonal states of the metric which define different distances in such way that the difference is smaller or equal than the limitation on space time measurement, so  $S_{l,l} > 0$ . We suggest that Bekenstein-Hawking entropy is  $S_{l,l}$ . Let us calculate  $S_{l,l}$  in the case of a black hole. Clearly, the main contribution to  $S_{l,l}$  is near the horizon ( $\rho \approx 1$ ) where the uncertainty is maximal,  $\Delta A \approx A$ . Thus we need to count the number of orthogonal states of the metric for which

$$0 \leq A(g_{\mu\nu}) \leq A_0, \quad (41)$$

where  $A_0 = 16\pi M^2$ . In the large  $M$  limit the gravitational radiation states

are plane waves, thus

$$h_{\mu\nu}(x, t) = \frac{e_{\mu\nu}}{\sqrt{\omega V}} \sum_k (a_k e^{ikx - wt} + a_k^\dagger e^{-ikx + wt}). \quad (42)$$

Where  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ,  $k_1 = \frac{n_1}{M}$ ,  $k_2 = \frac{n_2}{M}$ ,  $k_3 = n_3$  and  $w = \sqrt{k_1^2 + k_2^2 + k_3^2}$ .  $V$  is the volume of a thin sphere with width of the order 1. According to the postulates we can consider only  $k_i \lesssim 1$  hence the number of modes is of the order of  $A_0$  and  $w \approx 1$  so we get,

$$h_{\mu,\nu}(x, t) = \frac{e_{\mu,\nu}}{\sqrt{A_0}} \sum_k (a_k e^{ikx - wt} + a_k^\dagger e^{-ikx + wt}). \quad (43)$$

Now,

$$A(g_{\mu,\nu}) = \int \sqrt{g} dx_1 dx_2 \approx \int (1 + \frac{1}{2}h - \frac{1}{8}h^2) \quad (44)$$

Using Eq.(50) we obtain

$$A(g_{\mu,\nu}) \approx A_0 - \sum_{k_1, k_2} N_{k_1, k_2}. \quad (45)$$

Since the number of ways to distribute  $N$  quanta among  $M$  state is  $\frac{(N+M-1)!}{(N-1)!M!}$  and the number of mode is of the order of  $A_0$  we get

$$S_{l,l} = cA_0, \quad (46)$$

Since the whole discussion is based on qualitative arguments (the uncertainty principle,  $x_c \approx 1$ ), one should not expect to calculate  $c$ . Nevertheless, it is clear that  $c$  does not depend on the number of fields.

#### 4. Locality and causality



A physical theory is a theory which predicts the results of measurements. The following postulate is, therefore, somewhat natural.

*Postulate 3* The limitations on measurements in a physical theory follow naturally from the mathematical description of the theory.

The inspiration for the postulate is of course quantum mechanics, where the commutation relation  $[X, P] = i\hbar$  leads to the uncertainty relation  $\Delta X \Delta P \geq \frac{\hbar}{2}$ , which was originally found using measurement arguments. The measurement arguments alone do not prove that the mathematical description of a particle as a point in phase space is incorrect. However, even without considering other difficulties in the classical description, (black body radiation, photo-electric effect, stability of atoms, etc.) one can argue that it does not make sense to describe a particle as a point in phase space ( $\Delta X = \Delta P = 0$ ) and to claim an uncertainty relation  $\Delta X \Delta P \geq \frac{\hbar}{2}$  only when a measurement take place. Note that postulate 3 excludes information loss in principle.

According to postulate 3 the limitations on space time measurements in quantum gravity are due to the mathematical description of quantum gravity. Obviously, local quantum fields theory is not the proper mathematical description of quantum gravity in that case. Furthermore an unavoidable conclusion from Eq.(25, 35) and [23] is that the non-locality scale is not bounded. In other words, the non-locality scale is a function of the state of the system; in the case of a black hole near the horizon the non-locality scale is of the order of the size of the black hole radius! Therefore, postulate 2

cannot coexist with postulate 3. If postulate 2 is incorrect then in quantum gravity the non-locality scale depends on the state of the system! It might be very difficult to find the right mathematical tool to describe such a theory but it is hopefully possible. On the other hand if postulate 3 is incorrect then the whole description of physics by means of mathematics is meaningless since in that case there is no dictionary which connects between the mathematical description of physics and physics [23]. Therefore, in our opinion postulate 3 is more fundamental than postulate 1. In this paper we do not present the mathematical description which leads to an unbounded non-locality scale , but discuss qualitatively the possibility . We should remark that since the non-locality scale depends on the state of the system and it is one of the scales which define the state of the system it seems that finding the proper description means finding quantum gravity.

From Eq.(25) we find that the scale of non-locality is given by

$$d_r = 1 \tag{47}$$

$$d_{\varphi\theta} = \begin{cases} 1 & \rho > M \\ \frac{M}{\rho} & \rho < M \end{cases} \tag{48}$$

where  $d_r$  and  $d_{\varphi\theta}$  are the invariant non-local distance in the radial and angular direction respectively. Consistency of such a non-local theory implies non-locality in the time direction also. Otherwise the theory will suffer from acausal propagation at macroscopic scales. The non-locality in the time direction is then the time it takes for a light signal to travel from  $(r, \varphi, \theta)$  to

$(r + \Delta r, \varphi + \Delta \varphi, \theta + \Delta \theta)$ . In Rindler metric this gives

$$\Delta t = 8M \sinh^{-1} \frac{M}{2\rho^2} \quad (49)$$

In that case causality is preserve locally. Still, it does not mean that there is no causality violation since  $d_{\varphi\theta}$  depends on  $\rho$ , so there might be global violation of causality. Postulate 3 yields that the description of gravitation by means of local fields ,  $g_{\mu\nu}$  at a scale smaller than  $d$  is incorrect and that the correct description is such that the minimal length is given by Eq.(55, 56). In particular a particle which falls into a black hole is spread in the angular direction (since  $\theta$  and  $\varphi$  them selves are spread) according to Eq.(25).<sup>5</sup> fields at there is no is This spreading effect can causes a global violation of causality. Consider a signal emitted from point  $a$  ( $\rho = \rho_1, \theta = 0, \varphi = 0$ ) towards point  $b$  ( $\rho_1, 0, \varphi_1$ ) (see Figure 1). One can calculate  $t_{ab}$ , the minimal time needed by a classical perturbation to reach  $b$  from  $a$ . On the other hand due to the spreading effect one should consider another path: The signal is emitted from  $a$  toward  $c$  ( $\rho_0, 0, 0$ ) , ( $\rho_0 < \rho_1$ ) where it is spread with  $\Delta\varphi \geq \frac{1}{\rho_0}$ . Thus, if  $\rho_0 \leq \frac{1}{\varphi_1}$  then there is a finite probability for the particle to be emitted from  $d$  ( $\rho_0, 0, \varphi_1$ ) toward  $b$ . Naively the condition for causality is

$$t_{acdb} \geq t_{ab}. \quad (50)$$

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<sup>5</sup>This spreading effect obviously does not occur in the context of point like particle which is described by local field theory. In the context of string theory a similar effect was found by Susskind [25]. This is quite surprising since our calculations and postulates are rather model independent.

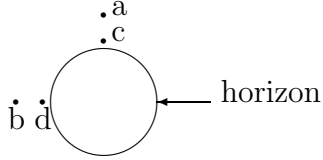


Figure 1: Causality implies that the nonlocality effects are such that  $t_{ab} - \Delta t \leq t_{ac} + t_{cd} + t_{db}$ .

However, we should also consider the non-locality in the  $t$  direction, meaning the uncertainty in  $t$ . One can detect causality violation only if

$$t_{acdb} < t_{ab} - \Delta t_{ab}, \quad (51)$$

since according to postulate 3 time is defined with minimal error  $\Delta t$ . In Rindler space it is easy to calculate  $t_{ab}$  and  $t_{ac}$ ,

$$t_{a,b} = 8M \sinh^{-1}\left(\frac{R}{2\rho_1}\right) \quad (52)$$

$$t_{ac} = t_{bd} = 4M \log\left(\frac{\rho_1}{\rho_2}\right) \quad (53)$$

where  $R$  is the distance between  $a$  and  $b$ . In order that path  $acdb$  will be possible we must have

$$R \leq \frac{M}{\rho_2} \quad (54)$$

thus

$$t_{ab} \leq 8M \sinh^{-1}\left(\frac{M}{2\rho_1\rho_2}\right) \quad (55)$$

Since  $\Delta R = \frac{M}{\rho_1}$  we obtain

$$\Delta t_{ab} = 8M \sinh^{-1}\left(\frac{M}{2\rho_1^2}\right). \quad (56)$$

So, the condition for causality is

$$8M \log\left(\frac{\rho_1}{\rho_2}\right) + 8M \sinh^{-1}\left(\frac{M}{2\rho_1^2}\right) \geq 8M \sinh^{-1}\left(\frac{M}{2\rho_1\rho_2}\right). \quad (57)$$

Fortunately this condition is satisfied. Equality is obtained for  $\rho_1 = M^a$  where  $a < \frac{1}{2}$ . Further more if the spreading effect were stronger (for example  $d = \frac{M}{\rho^c}$  where  $c > 1$ ) then causality would be violated. The spreading rate (Eq.(55)) is , therefore, the fastest rate consistent with causality.

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